Examining the Relationships among Dimensions of Engagement, Attribution for Failure, Self-Efficacy, and Mathematics Achievement

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 According to the National Assessment Governing Board’s (NAGB, 2014) technical report, the national mathematics score average for U.S. students has remained stagnate since 2003 with only 1 in 4 U.S. students reaching the baseline level 2 of mathematics proficiency. At the international level, in comparison with 34 participating members of the Organization for Economic Cooperation and Development (OECD) U.S. 15year-olds were ranked at the 27th percentile in mathematics (PISA 2012). This below average performance is likely to be detrimental to the prospects of job seekers, given that an estimated 50% of the jobs in the future will necessitate STEM skills; furthermore, according to the U.S. News/Ratheon STEM index (2014), jobs in STEM fields have increased by 30% over a 13-year period.

 Several factors have been identified as contributing to the decline in mathematics achievement including interest, ability, teacher support, student motivation, and mathematics engagement (Appleton & Lawrence, 2011; Green, Miller Crowson, Duke, & Akey, 2004; Lee & Stankov, 2013; Saritas & Akdemir 2009). This study focuses on mathematics engagement. It examines how attribution for failure, mathematics self-efficacy, and mathematics achievement relate to mathematics engagement, while controlling for students’ gender and socio-economic status. Attribution for failure is defined as judgments about prior experiences that exert influence on self-efficacy. Self-efficacy is students’ belief about their ability to organize and execute action for learning. Self-efficacy is positively correlated with achievement and engagement (Linnenbrink & Pintrich, 2003). Engagement is widely defined as active participation, positive and negative feelings regarding the activity at hand, and effortful investment (Christenson, Reschly, & Wylie, 2012). It is conceptualized as a multidimensional construct encompassing cognition, behaviors, and emotions (Frederick, Blumenfeld, & Paris, 2004). These dimensions are recognized to be inter-related, meaning one influences the other and vice-versa. Students who are engaged study with the mindset of mastering the learning, complete homework on time, pay attention and listen in class, enjoy learning, and look forward to lessons.

 This present study has a dual purpose in that it seeks to (1) examine the structure of the measurement model of engagement (see Appendix A1) and (2) explore the relationships among gender, social-economic status (SES), attribution for failure in mathematics, mathematics self-efficacy, mathematics engagement, and mathematics achievement. This undertaking is in response to a call by Christenson, Reschly, and Wylie (2012) to provide better “conceptual clarity and methodological rigor” to the study of engagement in order to advance the construct and inform educators of its usefulness for intervention programs. In addition, this study also recognizes the complexities involved in the learning process, and therefore has included factors that represent the intent and actions relevant for educational outcomes. This approach situates engagement as influenced by the student level factors of attribution for failure and self-efficacy.

 In the next section, the social cognitive theory framing this study is discussed. Following this is a general overview of the engagement, attribution for failure and self-efficacy constructs. Next, a review of the literature centered on engagement and achievement is presented. In the final section, the method of analysis for examining the measurement model of engagement and for exploring the relationships among gender, SES, attribution for failure, self-efficacy, mathematics engagement, and mathematics achievement is outlined. A model of the hypothesized relationships is presented(figure A2).

**Social-Cognitive Theory**

The social-cognitive theory underpins this study of engagement. This conceptualizes learners as active seekers of knowledge and holds that learning and performance are influenced by cognition and the environmental context in which it takes place (Schunk, 2012). Learners are assumed to be active participants who influence and are influenced by their unique environments. Moreover, individuals are seen as striving for a sense of agency through different types of learning: vicarious, symbolic and self-regulatory. Vicarious learning occurs through observation and imitation. Symbolic learning takes place through language, mathematics, science, reading, and writing. Self-regulatory processes emerge as individuals strive toward goals by choosing strategies, monitoring progress, and evaluating performance against goals.

 The reciprocity between learning, behaviors, and the social context is strongly reflected in social-cognitive research. According to this theory, engagement is shaped by the continuous interactions between cognitive, behavioral and environmental factors (Bandura, 1997; Schunk, 2012). The more widely accepted three-dimensional model is conceptualized as a encompassing cognition, behaviors, and emotions (Frederick et al., 2004); attribution is recognized as a key motivational variable seen as affecting achievement behaviors only through its influence on self-efficacy (Schunk, 2012); and self-efficacy is perceived as an influential cognitive belief with a powerful impact on individuals’ motivation, learning, self-regulation, and achievement (Bandura, 2005; Schunk 2012).

 In summary, this study is focused on Bandura’s social cognitive theory of psychological functioning to explore a model of engagement hypothesizing (1) attribution for failure as directly influencing (2) mathematics self-efficacy, the “essential [ingredient] for adaptive behavior” (Heider 1958**)** influencing both (3) mathematics engagement and (4) mathematics achievement (Bandura, 1977; Bandura & Schunk, 1981; Zimmerman, Bandura, & Martinez-Pons, 1992),while controlling for (5) gender and (6) social economic status (SES).

 In the next three sections, a brief description for the engagement, attribution, and self-efficacy constructs is provided, as well as how each is conceptualized within this study.

**Engagement**

Student engagement refers to the student’s active participation in academic and co-curricular or school-related activities, and commitment to educational goals and learning. Engaged students find learning meaningful, and are invested in their learning and future. It is a multidimensional construct that consists of behavioral (including academic), cognitive, and affective subtypes. Student engagement drives learning; requires energy and effort; is affected by multiple contextual influences; and can be achieved for all learners. (Christenson, Reschly, & Wylie, 2012)

 Understanding the influence of engagement on students’ mathematics achievement and behaviors is the central focus of this study. Given the number and types of definitions that exist in the current body of literature, it is important to begin this study of engagement with a clear conceptualization of what engagement is and what it is not. Engagement is a ‘messy’ construct evidenced by the varied conceptualizations and definitions found in the existing literature. (Brophy, Rashid, Rohrkemper, & Goldberger, 1983; Fredricks et al., 2004; Reschly, Huebner, Appleton, & Antaramian, 2008). Studies published in the past three decades give evidence to the complexity of the engagement construct. Frameworks and conceptualizations of the structure of engagement vary as a function of the grounding theory applied by researchers (Alexander, Entwisle, & Horsey, 1997; Blumenfeld & Meece, 1988; Brophy et. al. 1983; Finn 1989; Martin, 2007; Mosher & McGowan, 1985). As stated previously, this study of engagement is grounded in the social cognitive theory and as such it is focused on the individual learner and their active pursuit of mathematics success.

Early research situated engagement inside the school dropout literature, which defined it as participation in learning activities afforded (Mosher & McGowan, 1985). The dropout literature conceptualizes engagement as a protective factor for staying in school (Brophy, 1983). This literature found contextual factors, e.g., task variables and situational factors, to have a strong influence on behavioral engagement, which in turn influenced school dropout rates (Alexander et al., 1997; Mosher & McGowan, 1985; Wehlage, 1989). For the most part, early researchers approached engagement from the developmental perspective focusing on the observable behaviors of engagement: absenteeism, participation, and delinquency. It was conceptualized as having multiple determinants that were interactive, rather then additive or mediators, and included psychological factors such as intelligence, self-concept, aspirations, attitudinal aspects, valuing, and attribution beliefs (Mosher & McGowan, 1985).

 The view of engagement as a construct began to expanded as more research grounded in achievement motivation lent support for viewing engagement as influential for helping struggling students achieve (Blumenfeld, 1991). For example, research seeking to clarify the role of goal theory identified tasks and the environment as important influencers for the types of goals developed by learners (Blumenfeld, 1991; Blumenfeld et a., 1991). This line of research resulted in the identification of two indicators of engagement: goal orientation and the importance of valuing learning (Blumenfeld et al., 1991). Further advances of the engagement construct occurred as researchers began to examine engagement as an outcome of the school process and to conceptualize it as having components (Finn & Voelckl, 1993).

Research of the past ten years has mainly focused on understanding why engagement matters to achievement, often citing the seminal work of Fredricks, Blumenfeld, and Paris (2004), which conceptualized engagement as uniting 3 components that form the action of the motivational intention (Russell, Ainley, & Frydenberg, 2005). These are: (1) The cognitive dimension, defined as investment and effort toward mastering ideas and skills; (2) The behavioral dimension, reflected as participation; and (3) The emotional dimension, encompassing positive and negative reactions to learning.

 This study draws from the widely accepted Fredricks et al. (2004) demarcation of engagement into three dimensions. The engagement dimensions with factors derived from the PISA 2012 dataset is presented below.

**The Cognitive Dimension**

The cognitive dimension is defined as being personally invested and having a willingness to exert effort to learn mathematics (Fredricks & McColskey, 2012). It includes measures reflecting the latter definition: (1) work hard on mathematics homework, (2) be prepared for mathematics exams, (3) study hard for mathematics quizzes, (4) study until I understand mathematics, and (5) avoid distraction when studying.

**The Behavioral Dimension**

The behavioral dimension is defined as units of actions specific to mathematics achievement (Christenson et al., 2012; Fredricks et al., 2008). The 4 measures included in this dimension gauge to what degree students complete homework and participate in their mathematics class. These are: (1) I complete homework in time for my class; (2) I pay attention in mathematics class; (3) I listen in mathematics class; and (4) I seek explanations.

**The Emotional Dimension**

The emotional dimension is defined as student feelings toward learning mathematics. In this study 3 measures are employed that reflect student enjoyment: (1) I look forward to mathematics lessons; (2) I enjoy mathematics; and (3) I enjoy reading about mathematics.

 As a relative new field of study, engagement is yet to be clearly conceptualized or defined and therefore researchers have a responsibility to demonstrate that the structural model of engagement is a good fit with data. This study intends to add to the existing knowledge by presenting a structural model of engagement that is supported by data and can be used to predict students’ mathematics engagement.

**Attribution Theory**

Negative beliefs [constructed from experiences] trigger negative perceptions and emotions resulting in both nonfunctional thinking and behaviors. (McCombs & Marzano, 1990)

 The attribution theory is concerned with the causal reasons learners give for academic successes or failures. This attribution theory is important to the study of engagement because of the influence that judgments about prior experiences exert on self-efficacy and on each of its dimensions: cognitive, emotional and behavioral (Weiner, 2008). Learners who attribute their success or failure to internal and controllable causes are more inclined to feel efficacious about school and more likely to use self-regulatory processes (e.g., goal setting, strategy choices, progress monitoring, and self-evaluation) than their peers who claim internal and uncontrollable causes (e.g., ability as stable; luck or difficulty of subject) for their success or failure (Schunk, 1994). When learners recognize, appreciate, and understand their academic ability as malleable, they are able to engage in appropriate actions (e.g., goal setting) and assume a greater sense of control over academic behavior and motivation (McCombs & Marzano, 1990; Stewart, Latu, & Myers, 2010).

 According to Weiner (2010), causes for success and failure are represented along three dimensions: (1) internal or external to the individual; (2) stable or unstable over time; and, (3) controllable or uncontrollable by the individual. Effort is regarded as internal, controllable and unstable. Ability, on the other hand, is internal, uncontrollable and stable. Success attributed to stable causes results in higher student self-expectations than success attributed to unstable causes. A learner’s affect (e.g., pride or shame) is influenced by the attribution of success to internal (e.g., effort) or external (e.g., luck) causes. The choices of tasks, level of effort, persistence, and level of achievement depend on learners’ perception of how much control they have over academic outcomes.

 Dysfunctional attributional thinking erodes goal striving, the purpose or reasons that learners use to persist in the face of failure (Pintrich, 2000; Stewart et al., 2010). Therefore, consideration of a learner’s attributional state is key in understanding how to facilitate positive self-efficacy toward mathematics engagement and mathematics achievement.

**Self-Efficacy**

 Self-efficacy is a belief that one has the ability to execute the appropriate actions toward an outcome without serious doubts. Bandura (1977), a seminal figure in the self-efficacy literature, posited that self-efficacy determines the level of effort and the degree to which strategies will be employed.

 Current research maintains self-efficacy is highly influenced by the processes inside the self-regulation model of learning espoused by social-cognitive theorists (Zimmerman, Bandura, & Martinez-Pons, 1992). Enhancing mathematics self-efficacy results in improved homework behaviors, effective learning approaches, the ability to set appropriately challenging goals, and academic engagement (Kitsantas, Cheema, & Ware, 2011; Martin, 2012). Sources of self-efficacy include mastery experience, vicarious experience, verbal persuasion, and physiological states (Bandura, 1997).

Most studies have focused on investigating the powerful role self-efficacy plays in explaining achievement. More studies are needed to understand the relationship of self-efficacy to the variables of interest in this study: attribution for failure, engagement, and achievement.

 This present study proposes to further existing knowledge by testing a causal model showing self-efficacy as mediating the relationship between attribution for failure in mathematics and mathematics engagement/mathematics achievement. A more extensive review of the literature centered on engagement follows. While some literature directly deals with mathematics, the majority of the literature discusses engagement in more general terms.

**Engagement and Mathematics Achievement**

The multidimensional engagement construct offers a robust approach for studying the complexity and intricacies for how learning happens (Chase, Hilliard, Geldhof, Warren, & Lerner, 2014; Fredricks et al. 2004; Ladd & Dinella, 2009; Hampden-Thompson & Bennett, 2014; Lam, S., Wong, B., Yang, H., & Liu, Y., 2012; Martin, Way, Bobis, & Anderson, 2015; Rimm-Kaufman, Baroody, Larsen, Curby, & Abry, 2015; Sciarra & Seirup, 2008). Engagement has the potential to capture what it takes to achieve, yet little research exists employing more then two of the dimensions together. One such study, was conducted by Sciarra and Seirup (2008) using the Educational Longitudinal Study ( ELS, 2002/2004) to investigate the unique variance accounted for by engagement on mathematics scores across 5 racial groups (N= 11, 388; Indian, Asian, Black, Latino, and White). Engagement was conceptualized as a three-dimensional construct related to investment in learning, participation in school and learning activities, feelings about school, and caring about school. The cognitive engagement was operationalized with 10 measures reflecting students’ commitment to learning, assigned importance of good grades, perseverance, homework completion, and time on homework. Behavior engagement was operationalized as having 3 components measured with 14 items related to learning, compliance, and participation (frequency of lateness, cutting, absences, disruptive/attentive behaviors, disciplinary actions, and time dedicated to extra activities). The emotional dimension of engagement was operationalized with 24 items reflecting quality of student-teacher/peer relationships, school safety, and harmony of racial groups.

 This study reported that a combination of forty-eight engagement variables representing cognitive engagement (commitment to learning, importance of grades, perseverance, homework completion/time), behavioral engagement (lateness, attendance, disruptive discipline, and extra curricular participation), and emotional engagement (teacher/student relations, peer relations, school safety, and harmony of races) significantly predicted mathematics scores for Latino (R2 = .07) and White (R2 = .14) students. However, for the white students the emotional engagement dimension had less impact than the other two dimensions. For the Asian (R2 = .11), Black (R2 = .07) and American Indian (R2 = .21) students, behavioral engagement and cognitive engagement emerged as significant predictors, while emotional engagement was not significant. Overall, this study showed cognitive engagement and behavioral engagement predicted mathematics achievement. A major limitation of this study is the broad measures used to construct the engagement factor. For instance, the outcome measure used was mathematics scores at the individual level, yet the factors that measured engagement were at the school level. This may account for the low loading on each of the three dimensions (ß range = .05 to .33). While this study showed the predictive value of the engagement construct for achievement, the low loadings indicate the structural model needs refinement. Convergent validity is essential when fitting a causal model.

 In a similar longitudinal study with a sample size of 710 students, 85% white, with a mean age of 15.7 and an average GPA of 3.53, Chase et al. (2014) conducted a multilevel analysis to explore the reciprocal relationship between engagement and school achievement across three years of high school. Engagement was conceptualized as having three dimensions. The cognitive dimension was measured by 4 items reflecting value education/learning and thoughts about learning); the behavioral dimension was measured with 5 items reflecting attendance and effort; and the emotional dimension was measured with 5 items reflecting sense of belonging. The findings showed that two of the strongest relationships appeared between grade 10 behavioral school engagement and grade 12 GPA (ß=.19) and grade 10 GPA on Grade 12 behavioral school engagement (ß=.21). In addition, all engagement dimensions showed auto-regressive relationships across grades; meaning, engagement is a “richer get rich” factor, i.e., if students exhibit engagement at one time period, it is likely they will exhibit engagement at a later period. These findings support the current study’s intention to investigate engagement as both an outcome process of contextual factors such as attribution for failure and self-efficacy, and as a predictor for future outcomes.

 While this study was able to show engagement’s critical role in predicting student outcomes, the operationalization of engagement encompassed too many variables that confounded the definition of the dimensions by overlapping motivational variables indicating intent with engagement variables signaling effortful action. For instance, cognition included valuing, a variable of intent rather than action. Moreover the emotional engagement variable, “sense of belonging,” is closely related to the drive theory of motivation that again is a source of motivation not engagement. Clearly more research is needed to show the predictive value of engagement with separation of motivational and engagement variables.

 In a different study that operationalized engagement as a two-dimensional construct, Ladd and Dinella (2009) sought to explain the additive features of engagement for 383 students’ achievement followed from kindergarten to grade 8. Engagement was conceptualized as processes that promote learning and achievement. Behavioral engagement was defined as involvement in classroom activities and adapting to classroom norms, while emotional engagement was described as receptiveness to school. Emotional engagement was an average score of 10 items from parent reports and 10 items from teacher reports that asked about school liking and avoidance. The behavioral engagement was measured with five items from teacher reports about classroom behaviors. The findings showed engagement correlated positively with achievement from grade 1 to 8. Moreover, earlier emotional and behavioral engagement predicted subsequent engagement and achievement. However, only behavioral engagement proved a consistent predictor of achievement across grades. Of note, Ladd and Dinella (2009) reported finding a difference by gender, ethnicity, and SES across the grades for engagement. Girls were significantly more behaviorally engaged in the primary school years then boys (grades 1 to 3), however, the strength of this correlation weakened in grades 4 to 8. As can be seen, this study more narrowly defined the components of engagement, and unlike the previous studies, the more narrow definition of emotional engagement produced significant observable data. Whether this narrower operationalization of the emotional engagement proved to help the predictive value of the factor needs to be further explored with students in higher grades.

 From the previous study, it is evident that student level factors of gender, ethnicity, and socioeconomic status add to the understanding engagement and achievement. These factors help disentangle what matters for engagement and for whom engagement intervention matters most. In support, Hampden-Thompson and Bennett (2014) presented data from the PISA 2006 providing evidence of the association among engagement (enjoyment, future orientation, affect instrumental motivation, future orientation to science), student factors (gender and SES), class factors (teacher support, hands-on lessons, applicable lessons, and student investment), and school factors (class size, selective schools, ration student/teacher, shortage of teachers, and science activities available). This study conceptualized engagement as a three-dimensional construct representing psychological investment in learning, positive and negative emotions about learning and school, and effortful actions. Only two engagement dimensions were included in this study. The emotional engagement dimension was defined as enjoyment of science and the cognitive engagement dimension was demarcated as instrumental motivation and future orientation to science. The data revealed that higher frequencies of debates/discussions, hands-on investigations, and application experiences predicted engagement (higher levels of enjoyment, future orientation and motivation toward achievement). In addition, the student factor of gender was found to be positively associated to engagement, with males more engaged than females. Other sources of engagement included the school factors of shortage of teachers and class size.

 This study is exemplary in that it shows the complexity that exists in learning; however the inclusion of motivational variables as indicators of cognitive engagement (interest) and emotional engagement (instrumental motivation and orientation to science) –as well as allowing variables (future orientation) to load on both factors of engagement– calls into question the structural validity of the model. Therefore, the present study proposes to narrow the variables to a clearly defined engagement construct in order to ensure the latent factor of mathematics engagement measures what it is suppose to measure, i.e., the components of engagement, not its causal factors.

 As discussed earlier, current engagement research is focused on achievement and examining the factors for changing the downward trajectories of achievement, i.e., “catch them before they fall.” Therefore, investigating the impact of student and contextual factors in learning is valuable information in understanding which factors to target in order to “reengage” students. For example, a recent study by Martin et al. (2015) found that student factors of self-efficacy and valuing explained the bulk of the variance in mathematics engagement shifts in the academic lives of students occurring at key transition points from grades 5 to 8. This study reporting on 1,661 Australian students revealed engagement’s strong auto-regressive effect; meaning prior engagement predicted future engagement in the middle years. Engagement was conceptualized as having three dimensions and was defined as active participation, positive and negative reactions to academic environment, and a willingness to invest effort. It was operationalized with measures such as planning, task management, and persistence for cognitive engagement. Class participation, homework completion and effort were measures of behavioral engagement. Emotional engagement was measured with enjoyment of math. Overall, declines in engagement in transition periods within grade 5 to 8 are better explained by student variables such as self-efficacy, valuing, and achievement, and to a lesser extent with the contextual factors for parent valuing and classroom climate. Of note, gender differences were not found in this sample of students. This is surprising, especially because most research in mathematics reports differences by gender; specifically females are consistently shown to lag behind males in mathematics self-efficacy, engagement, and achievement.

 Contrary to the gender findings by Martin et al. (2015), Rimm-Kaufman et al. (2015) report finding engagement to be predicted by gender in mathematics classrooms. In a sample of 385 fifth grade students in Virginia, girls were more cognitively and socially engaged than boys. These researchers conceptualized engagement as a four-dimensional construct that included behavioral, cognitive, emotional, and social engagement. The study reported data from observation, a teacher-reported behavioral dimension, and student-reported engagement. The observed behavioral dimension was defined as observable participation in learning opportunities, disruptive behavior, and self-reliance (time-sampling and global ratings). The teacher-reported behavioral dimension was measured with 8 items reflecting exertion in doing work, attention, and participation. Student-reported cognitive engagement was measured with 6 items reflecting effortful investment in mathematics learning. The student-reported engagement measures included six items for cognitive engagement (e.g., Today in math class I worked as hard as I could), five items for emotional engagement (e.g., Math class was fun today), and four items for the social engagement dimension (e.g., Today I talked about math to other kids in class).

 Influential contextual and student factor findings included higher levels of organization in the classroom and more emotional support from teacher. Higher levels of organization impacted behavioral engagement in a positive direction. Of note in this study is the interaction by gender findings. While girls, as reported earlier, were generally more engaged then boys, the cognitive and emotional engagement of boys increased as a function of increases in classroom organization. In addition, as instructional support increased, social engagement decreased for girls, but not for boys –meaning, girls participated less in peer-to-peer thinking about math as a result of more instruction. It is unclear what this association means, but clearly more studies are needed that investigate why this association is present for girls and not boys.

 Based on the empirical evidence to date, it would appear that engagement warrants further investigation (Christenson et. al., 2012). Most studies reported above have focused on examining only one or two dimensions related to achievement, more studies with the more prevalent three-dimensional model of engagement that includes cognition, behaviors and emotions. A critical first step is to lessen the “conceptual haziness” by clarifying what indicators best fit the engagement construct. Most of the studies presented showed satisfactory goodness-of-fit statistics but not consistent convergent validity. This present study attempts to establish a clear conceptualization of engagement by first reporting results from a confirmatory factor analysis (CFA) demonstrating that measures of engagement reflect goodness-of-fit, convergent and discriminant validity. Next, a structural equation model (SEM) will show how engagement functions as both a facilitator for mathematics achievement and as an outcome of influential factors such as mathematics attribution for failure discussed in the next section.

**Attributions for Failure, Engagement, and Achievement**

Attribution for failure is a motivational factor that exerts influence on engagement and achievement. Social cognitive theorists seeattribution for failure as being affected by experiences, social norms, and causal reasoning (Weiner, 2008). It is well known that prior experience shapes future striving (Bandura, 1977; Schunk, 1994; Zimmerman, Bandura, Martinez-Pons, 2008). Therefore, students prone to failure are vulnerable often displaying low self-efficacy, low motivation to engage meaningfully in their studies, and low achievement (Lam, Wong, Yang, & Liu, 2012; McClure, et al., 2011; Perry, Stupnisky, Chipperfield, & Weiner, 2013; Wolters, Fan, & Daugherty, 2013).

 Engagement and achievement striving are influenced by student’s attributions for failure. Attribution for failure can be both adaptive and maladaptive. Effort is considered an adaptive attribution for failure as it can be changed, is an internal cause, and can be controlled by an individual. On the other hand, attribution for failure to luck and/or task difficulty is considered maladaptive. In a recent study on attribution for failure, McClure et al. (2011) found that Grade 11 student’s attribution for best and worst marks was related to their motivation orientation and achievement. These researchers examined the relationship among attribution for failure, motivation orientation, and achievement of 1,533 New Zealand students. In this study, students were asked to rate their best and worst grades using the 7 causal attributions of *ability, effort, luck, task difficulty, teacher, peers,* and *family*. Overall, the results revealed that females tended to attribute their worst marks to *ability* (stable) and the *teacher* (external/uncontrollable), while males tended to attribute their worst marks to *luck* (external/uncontrollable). These maladaptive factors incite students’ feelings of having no control, thereby decreasing their efficacy to engage in learning. As for achievement, students who attributed failure to *effort* and *ability* performed better than students who attributed their worst marks to *ability, effort, luck, task difficulty*, and *teacher*. This study clearly showed the important link between attribution for failure, engagement, and achievement, as well as a difference in attributional thinking by gender.

 This relationship between attributions, engagement, and achievement is best revealed in attributional retraining studies, focused on shifting attributions for failure from the mindset that outcomes are uncontrollable, stable (not malleable), and externally determined to a mindset that one is capable of organizing and taking appropriate actions to attain a goal. For example, Perry et al. (2013) examined the association of maladaptive attributional beliefs (e.g., luck, ability, teacher, task difficulty) of first-year university students with their engagement and achievement. The study attempted to change the attribution for failure, achievement-related emotions, and achievement performance of 459 students enrolled in an entry-level psychology course. Participants were placed in either a *no-attribution retraining* group (No-AR) or in an *attribution retrainin*g group (AR). At time 1, the AR group received feedback on the first test of the semester and were encouraged to view failure with a mindset that it was within students’ control to change outcomes. The NO-AR group received no feedback or training at any time. At time 2, attribution and emotion measures were collected for all participants. At time 3 final grades were collected for the course and semester.

 The results revealed that achievement for low and average students in the AR group were higher than their peers in the No-AR group. The AR treatment (communicating beliefs that attributions for failure were internal, controllable and malleable) changed students’ motivation to engage in strategy use and invest more effort in their course work. In addition, the AR group showed an improvement in their final grades for the course over their No-AR peers. Furthermore, the AR group also attained higher semester GPAs than their No-AR peers. The link between attribution for failure and achievement-related emotion was inconclusive, possibly because it was measured with only one item. The present study further examines the link between attribution for failure, engagement, and achievement in mathematics with at least three items measuring each dimension of engagement.

 In another study investigating the link between attribution beliefs, engagement, and outcomes, Lam et al. (2012) position attribution as an antecedent of student engagement. In this study, involving 822 Chinese junior high school students, engagement was conceptualized as a three-dimensional construct. Each dimension included 12 items. The cognitive dimension was defined as students’ deep processing and was measured with items such as *when I study, I try to connect what I am learning with my own experience.* The behavioral dimension was conceptualized as effort in learning and involvement in school activities and was captured with questions such as *I try hard to do* well. Emotional engagement was conceptualized as liking for learning and liking for school and was measured with statements such as *I like what I’m learning in school*. In this study, attribution for failure to *effort* and *teacher* and*/or teacher strategy* was strongly associated with student engagement. Attribution to *effort* had a positive relationship with engagement, while attribution to *teacher* and/or *teaching strategy* had a negative correlation. While these findings support the model proposed in this study, its operationalization of engagement is hazy. For instance, the behavioral dimension is characterized as both effort in learning and involvement in school. Combining these two measures seems to be confounding data, as effort seems less a measure of behavior than school involvement. Furthermore, the behavioral measure of engagement, *I try hard to do* well, is too similar to the cognitive engagement measure of *when I study, I try to connect what I am learning with my own experience.* This overlap in the measures causes confusion as to whether the data is related to motivation (the intent) or engagement (the actions of learning). A clear demarcation between motivation and engagement is needed to better understand what is and is not considered engagement.

 A study by Wolters, Fan, and Daugherty (2013) adds to the evidence that attributional beliefs matter for engagement and achievement. Using a repeated measures analysis to examine the responses of 224 high school students from grades 9 to 12, these researchers showed that attribution for failure significantly predicted engagement. In addition, the study revealed a statistically significant relationship exits between causal attributions, self-efficacy, and measures of cognitive and behavioral engagement. Behavioral engagement was measured with 16 items reflecting choice, effort, persistence, and procrastination. Cognitive engagement was measured with 17 items reflecting cognitive strategies (linked to rehearsal, elaboration, and organization) and metacognitive strategies (linked to self-regulatory learning). The results revealed that attribution for failure to *ability* resulted in lower levels of cognitive engagement, while attribution for failure to *context* resulted in higher levels of cognitive engagement, specifically strategy use. Moreover, attribution for failure to *effort* predicted behavioral engagement. Of note, however, is the inability of attribution beliefs to contribute significantly to student performance outcome. This, explains Wolters et al. (2012), is consistent with Weiner’s (2005) view that attribution is more closely tied to psychological and behavioral variables than to distant outcomes.

 Another interesting result was the pattern suggesting self-efficacy as a potential mediator between attribution and performance. In the repeated analysis, self-efficacy emerged as a significant predictor of metacognitive strategies only after the addition of attribution beliefs. The latter result may indicate a reciprocal relationship between attribution beliefs and self-efficacy. According to Wolters et al., (2013) this is a reasonable contention given the results found; however, further investigation is needed.

 Based on the studies reviewed above there is sufficient support for considering attribution for failure an important influencer of engagement and achievement. Furthermore, the studies provide evidence of the viability of the present study’s hypothesized path model situating attribution for failure as an influencer on mathematics achievement through mathematics self-efficacy and mathematics engagement.

**Self-Efficacy, Engagement, and Achievement**

 Self-efficacy is one way to assess one’s motivation. It is the intent preceding the action of engagement leading to achievement. Numerous studies have found that self-efficacy determines students’ level of engagement and successful outcomes (Diseth, 2011; Green et al., 2004; Komarraju & Nadler, 2013; Martin et al., 2005**;** Ouweneel & Schaufeli, 2013; Wolters et al., 2013). However, the strength of self-efficacy on future action depends on previous experience of success and failure (Bandura 1977, 1997; Bandura & Schunk, 1981; Komarraju & Nadler, 2013; Linnenbrink & Pintrich, 2015). There is a scarcity of studies relating self-efficacy to engagement’s three dimensions of cognitive, behavior, and emotion in unison. Therefore, the evidence presented for how engagement and self-efficacy are linked includes investigations that examine one, two, or three-dimensional models of engagement.

 For instance, Green et al. (2004) conducted a study to test a causal model examining the impact of self-efficacy, perceived instrumentality, and goals on cognitive engagement and achievement. This study focused on the impact of self-efficacy, perceived instrumentality, and goals on the cognitive engagement and achievement of 220 high school students enrolled in English classes. Cognitive engagement was conceptualized as one dimension of a tripartite engagement construct that included behavior and emotion. Cognitive engagement was defined as the use of varied types of cognitive and metacognitive strategies that involved deep meaningful processing of information, enhancing memory and retrieval for learning. Green et al. examined path analysis data testing the causal ordering of mastery focus, autonomy support, non-competitive evaluation, instrumental motivation, and achievement in predicting cognitive engagement, self-efficacy, and achievement of goals. The results showed that self-efficacy (ß = .14) predicted cognitive engagement. Moreover, self-efficacy (ß = .38) and engagement (ß=. 15) together positively predicted achievement (grade percentage), and were the only variables with direct paths to achievement. This study supports the notion of a causal model that includes self-efficacy predicting engagement and achievement. What remains to be explored is a model that includes self-efficacy with more than one engagement dimension.

 In another study involving 177 first-year students at a Norwegian University enrolled in an introductory psychology course, Diseth (2011) used confirmatory factor analysis and structural equation modeling to investigate the relationship among prior achievement, self-efficacy, goal orientation, learning strategies (deep and surface level), and subsequent achievement. Like Green et al., (2004), Diseth (2011) focused only on cognitive engagement, defined as deep strategy use. This study found that self-efficacy positively predicted cognitive engagement and achievement. Moreover, self-efficacy was found to mediate the relationships between prior academic performance and engagement. In addition, Diseth reported a partial mediation effect, in which self-efficacy predicts engagement, which in turn predicts achievement. The findings confirm prior research showing the strong relationship between self-efficacy and engagement, and lend further support to the present study’s hypothesized causal ordering, e.g., mathematics attribution for failure (judgments about ‘prior experience’) predicting mathematics self-efficacy, predicting mathematics engagement, predicting mathematics achievement.

 In support of the proposed causal ordering above, Wolters et al. (2013) conducted a multiple regression analysis with the factors of goal orientation (mastery, performance-avoidance, performance-approach), self-efficacy beliefs, attribution beliefs, and engagement. Self-efficacy emerged as the strongest predictor of mathematics achievement for 224 high school students enrolled in mathematics courses. In this study, engagement was conceptualized as having three dimensions, but only the cognitive and behavioral engagement dimensions were included. The behavioral dimension was measured by instrumental value, effort, persistence, and procrastination, while the cognitive dimension was measured with the use of cognitive and metacognitive strategies. Wolters et al. (2013) reported that self-efficacy positively predicted both cognitive and behavioral engagement, and moreover, as reported earlier, self-efficacy emerged as the best predictor of achievement. Of note was the relationship that surfaced between self-efficacy and attribution for failure in mathematics. Specifically, when achievement goals and self-efficacy were together in a multilevel model, self-efficacy did not predict cognitive engagement. However, self-efficacy emerged as a significant predictor after attribution for failure entered the model. This pattern suggests that more investigation, like the present study is needed to further explore the link between students’ self-efficacy and attribution as related to students’ mathematics engagement and mathematics achievement.

 Komarraju and Nadler (2013) offer more insight into the potential of self-efficacy for predicting engagement and achievement with results from a correlation and hierarchical regression analysis. In this study self-efficacy, cognitive and metacognitive learning strategies (rehearsal, elaboration, organization, critical thinking, and self-regulation), resource management strategies (time and study environment, effort regulation, peer learning and help-seeking), and GPA are examined for 257 college students enrolled in an introductory psychology course. These researchers found that self-efficacy and engagement predicted 18% of the variance in GPA. The two-dimensional model of engagement used in this study consisted of cognitive engagement, defined by cognitive and metacognitive strategies, and behavioral engagement, defined by time on studies, study environment, effort regulation, peer learning, and help-seeking. The results showed GPA was positively correlated to self-efficacy, cognitive engagement, and behavioral engagement. Specifically, in block 1 of the step-wise regression analysis, self-efficacy (ß=. 30) emerged as the major predictor of GPA from among intrinsic and extrinsic motivation, value, control of learning, and low-test anxiety. In block 2, none of the cognitive engagement measures emerged as significant contributors to the variance of GPA. Conversely, in block 3, effort regulation and help-seeking were revealed as significant predictors of GPA. Of note was the mediation effect of effort regulation between self-efficacy and GPA. This study supports existing evidence that self-efficacy plays a major role in explaining achievement outcomes; moreover, it reveals self-efficacy’s mediating role as hypothesized in the present study’s causal ordering.

 Similar results linking self-efficacy, engagement and achievement levels were reported by Ouweneel et al. (2013). These researchers presented the results of two studies exploring how self-efficacy levels covary with engagement and performance. In these studies, engagement was conceptualized as a three-dimensional construct with an emphasis on the study engagement dimension. Study engagement was defined with measures reflecting cognitive engagement, which asked students about their effort, e.g., “when I’m doing my work as a student, I feel bursting with energy.” Study 1 included a sample of 335 university students (20 year-olds) who were asked about their self-efficacy levels over a period of 3 months, while controlling for age, years of study, and gender. Participants were categorized into four groups according to their self-reports about their self-efficacy at time 1 (T1) and time 2 (T2). The categories included the (1) the stability low group, which started with low self-efficacy at T1 and ended with low self-efficacy: (2) the increase group, which started with low self-efficacy at T1 that increased at T2; (3) the decrease group whose self-efficacy decreased at T2 from their already low self-efficacy levels at T1; and (4) the stability high group starting and ending with high self-efficacy. No significant difference was found for self-efficacy and engagement levels by age, years of study, or gender. Other findings revealed that changes in self-efficacy aligned with changes in engagement, and not with changes in performance outcomes (GPA). These researchers concluded that a causal path that places self-efficacy as firstly related with engagement and then with achievement could explain the relationship link between changes in engagement and self-efficacy. This notion seems justified as no link was found between levels of change in self-efficacy and performance. However, another explanation could be that self-efficacy in this study was not domain specific, e.g., these measures might not apply to mathematics. Self-efficacy has been shown to be a better predictor within specified domains rather than in general settings (Bandura, 1986).

 To document the causal assumption of study 1 Ouweneel et al. (2013) conducted a second study to explore the relationship among self-efficacy, engagement, and performance. Study 2 reported on data collected from 91 participants with a mean age of 20 years placed in one of 3 conditions: (1) positive feedback, (2) negative feedback, and (3) no feedback (control group). This study sought to test changes in self-efficacy, engagement and performance, while controlling for age, years of study, and gender. In this study groups were given IQ tasks related to spatial awareness, consisting of 15 questions at T1 and T2. In between T1 and T2, participants were presented with “bogus” positive, “bogus” negative, or “no” feedback with the aim of manipulating their self-efficacy. Positive feedback included a phrase at the end of T1 that stated, “Congratulations! Your IQ score belongs to the top 10% of participants so far.” The negative feedback statement included, “Unfortunately, your IQ score belongs to the lowest 10% of participants so far.” The control group was given no influencing statements.

 In this second study, the engagement measure was changed to reflect a change in task specificity, “I felt energetic when I carried out the task.” The results revealed no difference by age, years of study or gender. Interesting were the results of the MANOVA and post-hoc tests. In all 3 conditions a main effect was found for time on engagement and performance. A univariate analysis, however, indicated that the performance increase for the positive group was statistically significantly, but not engagement. In the negative group, the post-hoc test showed that both engagement and achievement decreased at a statistically significant level. The control group showed a significant decrease in engagement, but performance remained stable. These results show how manipulation of self-efficacy changes performance outcomes and supports the importance of including self-efficacy in the present.

 In a more recent longitudinal multilevel analysis, Martin et al., (2015) present still more evidence supporting the findings that self-efficacy is an important predictor of engagement and achievement. Specifically, Martin et al., (2015) sought to explain how student variables (self-efficacy and valuing), home variables (parent valuing and computer accessibility), classroom variables (average achievement and perceived climate), and school factors (socio-economic status and ethnic composition) influence mathematics engagement. They studied changes occurring at 3 key school-year transitions of 1,601 catholic school students in grades 5 through 8. This study showed that self-efficacy predicted mathematics engagement shifts and achievement across grades 5 to 8. Engagement was conceptualized as a three-dimensional construct including cognitive, behavioral, and emotional engagement. Cognitive engagement was defined as a willingness to invest effort and was measured with variables such as planning, task management, persistence, self-handicapping, and disengagement. Behavioral engagement was defined by active participation and was measured with variables such as class participation, homework completion, and effort (the extent and quality of homework). Emotional engagement was termed “affect” defined as the positive and negative emotional reactions to the academic environment and measured as enjoyment of mathematics. Consistent with other studies, Martin et al. (2015) recognized self-efficacy as a relevant influencer of students’ mathematics engagement. Further, key school-year transition effects on mathematics engagement were only found significant when the student, home, class, and school variables were not entered into the analysis. When the latter factors were subsequently entered, however, self-efficacy predicted the major share of the variance for all measures of mathematics engagement, except homework completion; here self-efficacy was the second best predictor after class-average homework. While the sample in this study was large enough to provide power to the study, it is important to test these results utilizing a U.S. sample. Further, it is important to account for SES as this has been shown to account for differences in achievement (Martin, Liem, Mok, & Xu, 2012).

 In summary, these studies link self-efficacy beliefs to engagement (cognitive, behavioral and emotional) and achievement; and confirm self-efficacy to be a major motivational variable that signals students’ intent to engage. This present study seeks to further this knowledge in a number of ways. First, it will provide a test of the causal ordering that places self-efficacy as a mediator between attribution for failure and achievement, as well as mediating the relationship between attribution for failure and engagement. Second, it seeks to establish a structural model of engagement based on a well-defined conceptualization of the variables in accordance with the social-cognitive theory of motivation and achievement. Third, this study seeks to replicate previous findings demonstrating the impact on mathematics achievement for attribution for failure in mathematics, mathematics self-efficacy, and mathematics engagement (see Appendix B). The proposed causal model, in its entirety, has not previously been tested. The findings will enhance our understanding by answering the following research questions:

1. What variables adequately represent the latent construct of engagement?
2. How does attribution for failure in mathematics relate to mathematics self-efficacy?
3. How much of the variance in mathematics engagement is accounted for by attribution for failure in mathematics and mathematics self-efficacy?
4. How much of the variance in mathematics achievement is accounted for by mathematics engagement?
	1. What is the unique contribution of each dimension to mathematics achievement
	2. Is there a difference by gender in the unique contribution of each dimension?

**Methods**

**Data Sources**

 The data comes from the 2012 Program for International student Assessment (PISA) that allows countries to compare learning outcomes of high school students. In the United States the National Center for Educational Statistics (NCES) conducted the surveys. This current year constitutes the PISA’s fifth cycle of data gathering. Each PISA data collection cycle assesses the three domains of mathematics, science, and reading literacy every 3 years. One domain rotates as the central focus in the PISA data collection. For 2012, mathematics was the expanded focus.

 The PISA national school sample consisted of 162 schools, an increase from the requirement of 150. This was done to offset non-response. The PISA requires a national sample standard of 35 to 50 per school. The survey was administered to 50 students: 42 used paper and pencil and 8 used computerized version. The survey lasts a total of two hours for each student, with two-thirds of the testing time being dedicated to mathematics, the major focus for year 2012.

 The data collected met international rate standards of 35-50 students, as well as the statistical standards of the National center for Education Statistics (NCES) of the U.S. Department of Education, 80 percent of schools and 85 percent of students. The Australian Council for Educational Research (ACER) selected the U.S. school sample for PISA 2012.

 The U.S. PISA sample was stratified into eight groups by school (public or private) and regions (Northeast, Central, West, Southeast). This frame was further stratified by categorical variables such as (1) grade range of the school (five categories), (2) location (city, suburb, town, rural); (3) ethnic diversity of above or below 15 percent (Black, Hispanic, Asian, Native Hawaiian/Pacific Islander, and American Indian/Alaska Native students, (4) gender (mostly female [percent female >= 95 percent], mostly male [percent female < 5 percent]; and other), and (5) state.

 The assessment instrument was developed by international experts and was reviewed by representatives from each of the education systems expected to participate in the PISA. All members of the education systems field-tested the 222 assessments items (85 mathematics, 44 reading, 53 science, and 40 financial literacy). The PISA consisted of a paper-based assessment with a two-hour administration time that provided achievement scores for students’ mathematics, science and reading literacy. An additional student questionnaire was completed in approximately 30 minutes that was comprised of items asking about student background, attitudes towards mathematics, and learning strategies. Finally, school principals were asked to complete a 30-minute school questionnaire describing the school’s demographics and learning environment. This present study only focuses on student response about their engagement in mathematics and their achievement mathematics achievement scores.

**Participants**

 The U.S. PISA 2012 national student sample consists of 4,720 students from 162 schools. After a preliminary analysis fifteen cases were deleted that showed irregular patterns in response. The sample was further reduced due to missing data. This was done using the listwise deletion method. The final analysis included 793 fifteen-year olds (M=15.8 years, SD = .29), 56% males and 44% females, enrolled in grades 9 to 11 (Mean = 10.05, SD = .525) from 156 schools.

**Measures**

 **Mathematics achievement.** Mathematical proficiency was measured with 85 items that assessed mathematical knowledge within a total testing time of 40 minutes. Test questions represented a wide range of difficulty.

 Three types of question formats were used to assess mathematical proficiency: multiple choice, closed short answer, and open constructed-response. An example of a closed short answer question is:

“The Gotemba walking trail up Mount Fuji is about 9 kilometers (km) long. Walkers need to return from the 18 km walk by 8 pm. Toshi estimates that he can walk up the mountain at 1.5 kilometers per hour on average, and down at twice that speed. These speeds take into account meal breaks and rest times. Using Toshi’s estimated speeds, what is the latest time he can begin his walk so that he can return by 8 pm?(OECD, 2013).

 Plausible values were used to measure *mathematics achievement* in this present study*.* Plausible values represent individual student levels as not every student was administered the same questions (Wu, 2005). Plausible values ranged from 0 to 1,000 with a mean score of 483 (for OECD, M=500). For each student five plausible values were attained and represent the range of abilities that may exist in the general population. For this present study only one plausible value was used as working with one plausible value has been shown to adequately provide an unbiased estimate of population parameters (OECD, 2012). There is very little variation across the 5 plausible values provided by PISA. The difference between any two plausible values is no more than 1.32 points. (OECD, 2012) The reliability of the five plausible values was α =0.98.

 **Engagement.** The mathematics engagement variables are indices constructed for this study and tested for reliability using Chronbach’s alpha.

 ***Cognitive engagement.*** Cognitive engagement is defined as students’ effortful investment in mathematics learning. It includes 4 measures derived from students’ response about their level of agreement with the following statements: (1) When confronted with a problem, I do more than what is expected of me, (2) I study hard for mathematics quizzes, (3) I keep studying until I understand mathematics material. (4) I do mathematics more than 2 hours a day outside of school. A four-point scale was used by PISA. The scale was inverted for ease of analysis, for negative statements the scale was not inverted. The response categories of ‘strongly disagree,’ ‘disagree,’ ‘agree,’ and ‘strongly agree’ have a point value from 1 to 4 The score range is 4 to 16. The reliability for this scale was **α = 0.85.**

 ***Behavioral engagement*.** Cognitive engagement is defined as purposeful actions for mathematics learning. It includes 4 measures derived from students’ response about their level of agreement with the following statements: (1) I work hard on my mathematics homework, (2) I finish my homework in time for mathematics class, (3) I am prepared for my mathematics exams, and (4) I pay attention in mathematics class. A four-point scale was used by PISA. The scale was inverted for ease of analysis, for negative statements the scale was not inverted. The response categories of ‘strongly disagree,’ ‘disagree,’ ‘agree,’ and ‘strongly agree’ have a point value from 1 to 4. The score range is 4 to 16. The reliability for this scale was **α = 0.85.**

 ***Emotional engagement*.** Emotional engagement is defined as the positive and negative emotions about mathematics learning. It includes 4 measures derived from students’ response about their level of agreement with the following statements: (1) I enjoy reading about mathematics, (2) I look forward to my mathematics lessons, (3) I do mathematics because I enjoy it, and (4) I am interested in the things I learn in mathematics. A four-point scale was used by PISA. The scale was inverted for ease of analysis, for negative statements the scale was not inverted. The response categories of ‘strongly disagree,’ ‘disagree,’ ‘agree,’ and ‘strongly agree’ have a point value from 1 to 4. The score range is 4 to 16. The reliability for this scale was **α = 0.85.**

**Attribution to Failure in Mathematics.** The attribution to failure in mathematics is a PISA scaled index using a weighted likelihood estimate (WLE). Attribution to failure in mathematics was constructed using student responses from six questions on a 4-point scale with a response category of “Not at all likely,” “Very likely,” “Slightly likely,” and “Not at all likely.” The scale was inverted for ease of analysis. This variable measured students’ perceived self-responsibility for failing in mathematics using the following scenario, “Each week, your mathematics teacher gives a short quiz. Recently you have done badly on these quizzes. Today you trying to figure out why.” Responses about their level of agreement were measured with the following statement: (1) I’m not very good at solving mathematics problems, (2) My teacher did not explain the concepts well this week, (3) This week I made bad guesses on the quiz, (4) Sometimes the course material is too hard, (5) The teacher did not get students interested in the material, and (6) Sometimes I am just unlucky. The score range is 4 to 24. The reliability for this scale was **α =** .727.

 **Mathematics self-efficacy.** Mathematics self-efficacy is a PISA scaled index using a weighted likelihood estimate (WLE) that asked student to respond with “very confident,” “confident,” “not very confident,” “not at all confident” about their confidence level for doing a number of mathematics tasks. Responses about their level of confidence were measured with: (1) “Using a train schedule to figure out how long it would take to get from one place to another,” (2) “Calculating how much cheaper a TV would be after a 30% discount,” (3) “Calculating how many square meters of tiles you need to cover a floor,” (4) “Understanding graphs presented in newspapers,” (5) “Solving an equation like 3x + 5 = 17,” (6) “Finding the actual distance between two places on a map with a 1:10,000 scale,” (7) “Solving an equation like 2(X+3) = (x+3)(x-3),” and (8) “Calculating the petrol consumption rate of a car.” The score range is 4 to 32. The reliability for this scale was α = 0.852.

 **Covariate variables .**The analysis included gender and social and cultural status (ESCS), two student level factors used as covariate variables. The ESCS was captured by PISA using 3 components with responses to open-ended question by students about their mother and father’s occupation and educational level. In addition a question about their possessions at home asks students to respond (1) yes or (2) no to a list of items that are in their home: e.g., a desk to study at, a room of your own, a quiet place to study, etc.

**Data Analytic Approach**

A confirmatory factor analysis (CFA) and structural equation modeling (SEM) were conducted using AMOS 22.0. Maximum likelihood was the method of model estimation. A CFA was chosen as it is deemed the most powerful approach for assessing the measurement validity of a theoretical structural model. CFA accounts for measurement error and incremental assessment of how the data fits the model in accordance with factor loadings, variance, and measured variable uniqueness. SEM was used to assess the structural model validity and assess the predictive power of the latent construct derived from CFA. Acceptable model fit requires acceptable reliability for each scale ( ≥ .70), acceptable model fit (CMIN ≤ 5, TLI and CFI ≥ .90 and RMSEA ≤ .08), acceptable factor loadings for items (>.50), and acceptable correlation (r < .90) among the latent factors (Marsh, Hau, & Wen, 2004). This analysis was conducted in three phases: Phase I was a CFA assessing the goodness-of-fit of the theoretical model of mathematics engagement, Phase II included a SEM analysis to validate and examine the predictive power of the structural model derived from CFA, and in phase III gender differences were examined by creating groupings in the SEM analysis.

**Missing Data**

 Missing values and patterns were identified using SPSS analysis that included Little’s Missing Completely at Random (Little’s MCAR) test. Missing data was relatively high at 37%; therefore the listwise deletion method was deemed appropriate to handle missing data.

**Data Preparation**

 An initial examination of the PISA (2012) dataset was conducted. The engagement construct was derived from 16 single items. All variables were reverse coded such that higher values reflect higher favorable responses.

**Educational Implications**

 This study provides information about the level of mathematics engagement of ​United States ​students​ as of 2012,​ ​ exploring the indicators and facilitators important for ​this ​engagement. ​ ​Examination​ ​of mathematics engagement​ indicators​ ​included building a confirmatory factor analysis (CFA) model to aid in clarifying the conceptualization of the engagement construct. ​ ​To ensure the model was theoretically sound, it was important ​to narrowly define the three dimensions of engagement​ and to ensure item loadings were a good-fit to the constructed model. ​Engagement was framed with a social-cognitive perspective, ​placing importance on student agency, an underemphasized student responsibility.​ ​ Therefore, this study focused on explaining how students’ beliefs about prior experiences shape​d​ their thoughts about their ​own ​capability to meaningfully​ ​engage in order to achieve​ academically. Structural equation modeling (SEM) was chosen as the best tool to assist in examining the proposed causal ordering.​ ​Overall, this study provided information that helps in identifying and understanding the ​factors that ​actually ​matter for engagement, ​and how to facilitate targeted and ‘on-time’ interventions to foster ​both​ engagement and achievement.

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**Appendix A1**

Figure 1 Multidimensional Engagement Construct



Note. A 3-factor model of engagement: *Cognitive engagement*: (1) When confronted with a problem, I do more than what is expected of me, (2) I study hard for mathematics quizzes, (3) I keep studying until I understand mathematics material. (4) I do mathematics more than 2 hours a day outside of school. *Behavioral engagement*: (1) I work hard on my mathematics homework, (2) I finish my homework in time for mathematics class, (3) I am prepared for my mathematics exams, and (4) I pay attention in mathematics class. *Emotional*: (1) I enjoy reading about mathematics, (2) I look forward to my mathematics lessons, (3) I do mathematics because I enjoy it, and (4) I am interested in the things I learn in mathematics.

Appendix A2

Figure 2 Engagement Predicting Mathematics achievement



Note. A model of the hypothesized relationships: attribution for failure influences self-efficacy, which in turn influences engagement leading to mathematics outcomes. In this model engagement is presented as both a mediator between self-efficacy and mathematics outcomes, as well as an outcome of attribution for failure through self-efficacy. Covariates of gender and SES are included in the model